3.15
Carrier Drift, Diffusion and R&G
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Reference: Pierret, chapter 3.

Electron and holes can drift, diffuse, and undergo generation and recombination (R&G).

Drift:

thermal velocity $1/2\ mv^2_{thermal} = 3/2\ kT$

drift velocity, $v_d = \mu \mathbf{E}$ (μ = mobility, $\mathbf{E}$ = field)

Current density (electrons) $J = n\ e\ v_d$

Current density (electrons & holes) $J = e\ (n\ \mu_n + p\ \mu_h)\mathbf{E}$

Conductivity $\sigma = J/\mathbf{E} = e\ (n\ \mu_n + p\ \mu_h)$

Magnitude of mobility (cm^2/Vs)

	μ_n	μ_h	
Si	1500	450	
Ge	3900	1900	
Ag	50	-	
GaAs	8500	400	

Time between collisions is τ $\mu = e\tau/m^*$

Distance between collisions is l $l = \tau v_{thermal}$

Diffusion

$J = eD_n\ \nabla n + eD_p\ \nabla p$

Derivation of the Einstein relation: $D_n/\mu_n = kT/e$

typical D_n in Si is 40 cm^2/s

Carrier R&G

Mechanisms: band-to-band (direct)
RG centers or traps (indirect)

Thermal R and G at equilibrium: R = G

expect $R = G = rnp = r\ n_i^2$ r = rate constant

Shining light, etc. on the semiconductor causes additional R. These excess carriers n_l and p_l ($n_l = p_l$)decay once the light is turned off.

Illuminated: n-type material

$n = N_D + n_l \quad \sim N_D$

$p = n_i^2/N_D + p_l \quad \sim p_l$

net rate of change of carriers = $R - G = rnp - r\, n_i^2$

the rate of recombination of the minority carriers is

$-dp/dt = r(N_D p - n_i^2)$ but $n_i^2 = N_D(p - p_l)$

$-dp/dt = rN_D(p - p + p_l) = rN_D p_l$

This has a solution $p_l = p_{l,\,t=0} \exp(-t/\tau_p)$, where $\tau_p = 1/rN_D$ = minority carrier lifetime.

<u>Example of Carrier Action – Formal solution</u>

A piece of p-type Si is illuminated at one end; how does the carrier concentration vary with depth x?

$dn/dt = dn/dt_{drift} + dn/dt_{diffn} + dn/dt_{thermal\ RG} + dn/dt_{other\ RG}$

$= 0$ at steady state

$n = n_p + n_l$ where $n_p = n_i^2/N_A$

Inside the material there is only thermal R&G:

$G_{thermal} = rn_i^2 = r\, n_p\, N_A$

$R_{thermal} = rnp \sim r\, n_l\, N_A$

$R - G = r\, N_A(n_l - n_p) \sim r\, N_A n_l = n_l/\tau_n$

In the steady state,

$dn/dt = dn/dt_{diffn} - (R - G) = 0$

$dn/dt = 1/e\ \nabla J_{diffn} - (R - G) = 0$

$d^2n_l/dx^2 = n_l/\tau_n D_n$

(since $dn/dt_{diffn} = 1/e\ \nabla J_{diffn} = D_n d^2n/dx^2$ from Fick's law)

solution: $n_l = n_{l,\,x=0} \exp(-x/\sqrt{\tau_n D_n})$

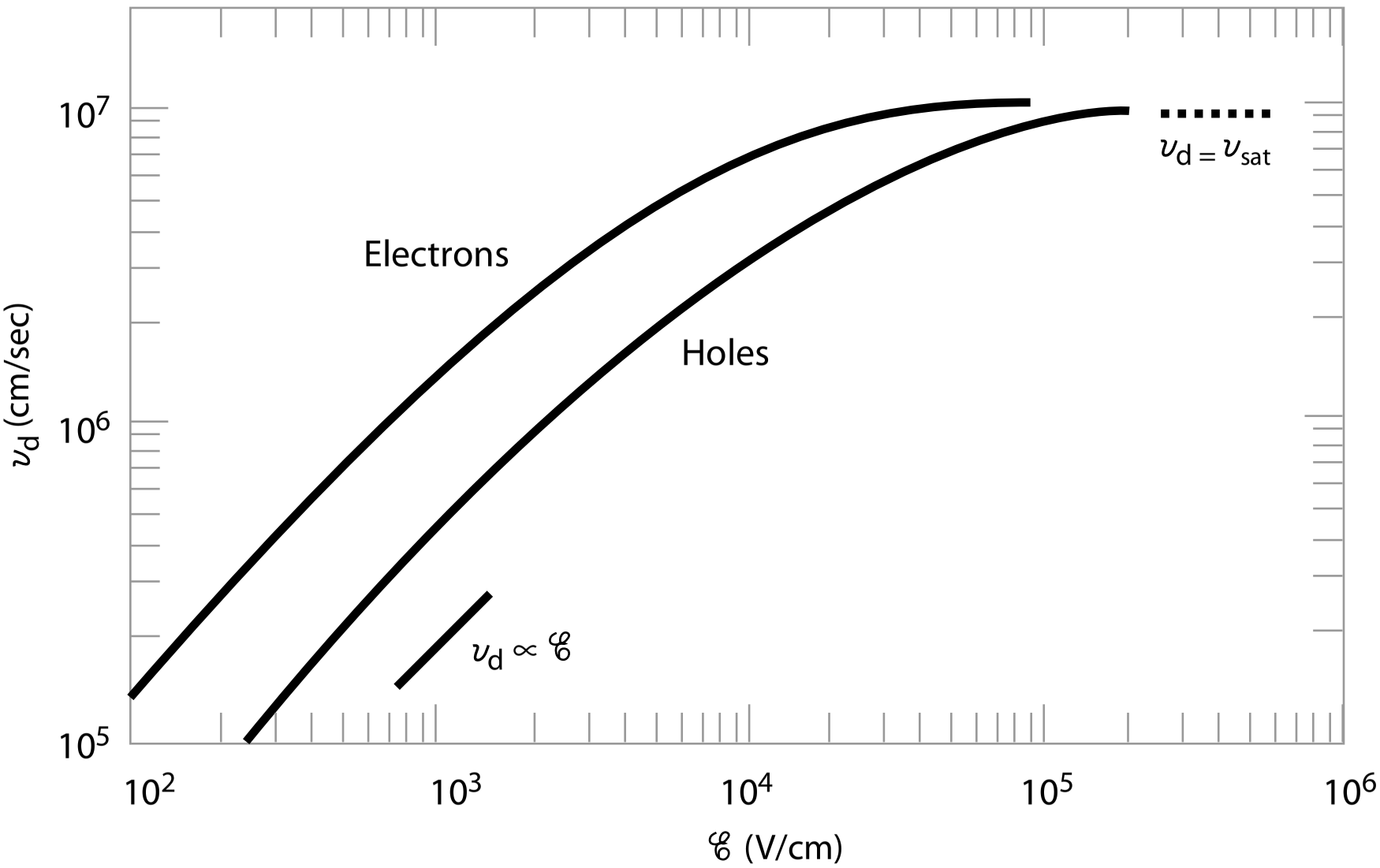
Generally, if excess carrier concentrations are written as n_l or p_l, then

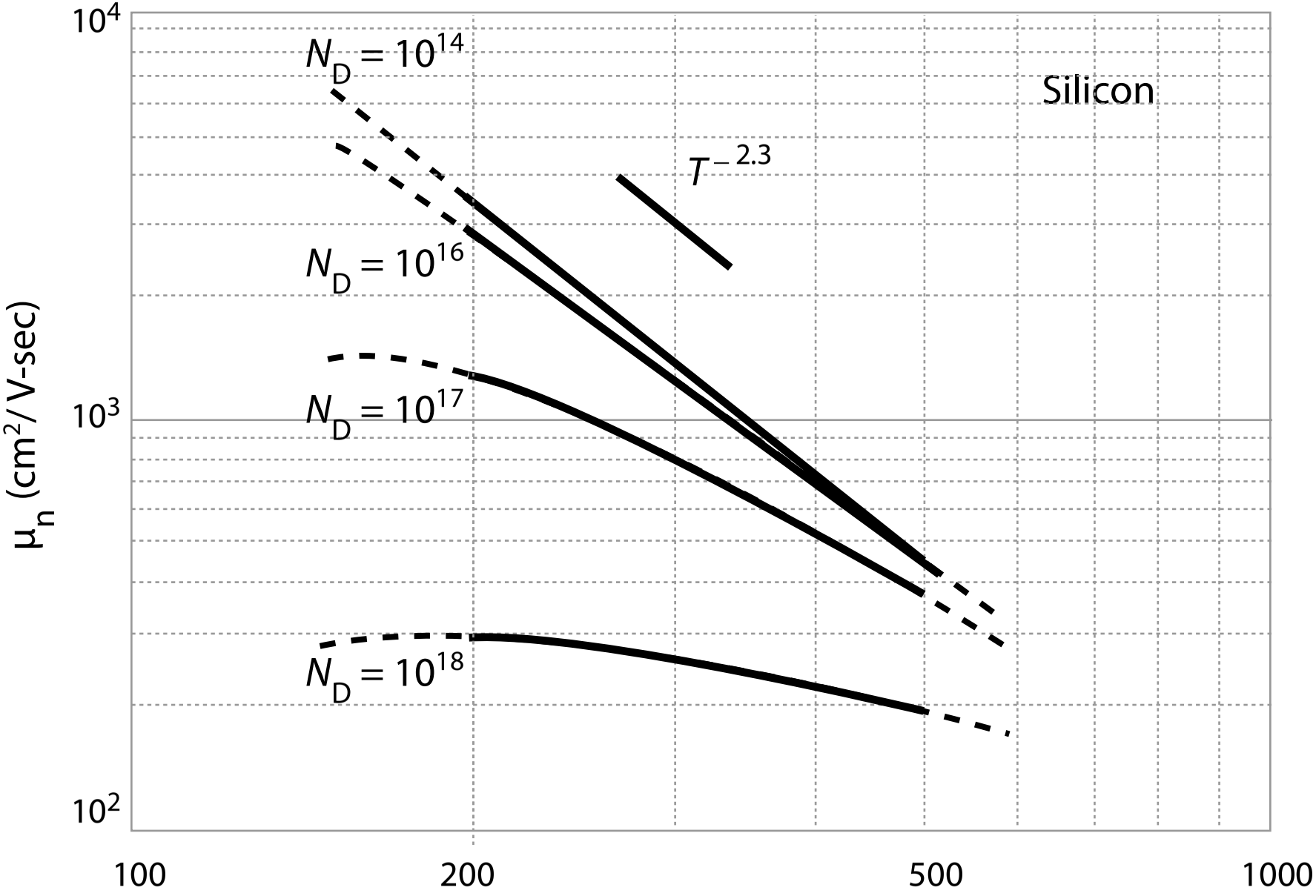
$dn/dt_{thermal} = -\, n_l/\tau_n$ or $dp/dt_{thermal} = -\, p_l/\tau_p$

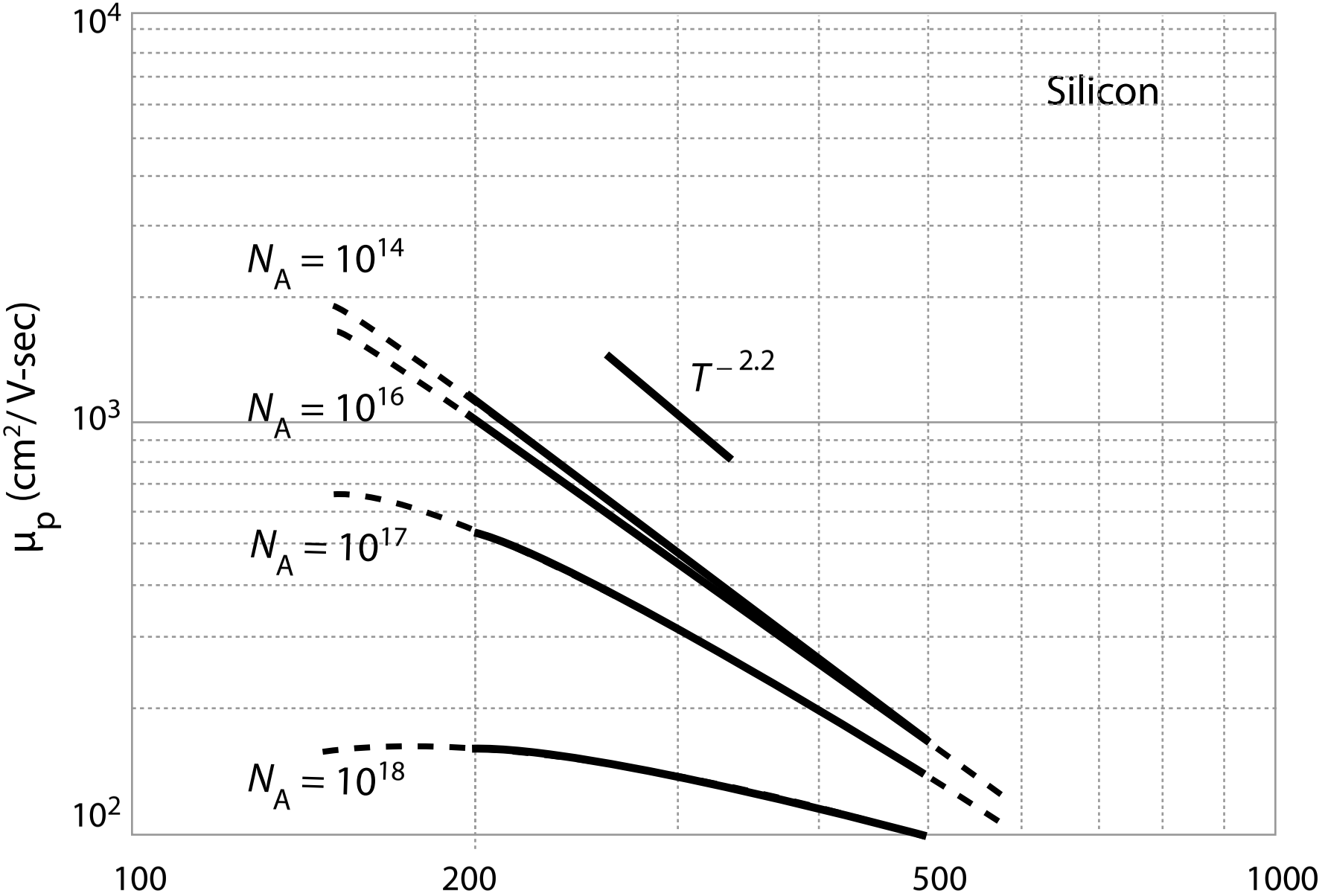
Minority carrier lifetimes are $\tau_n = 1/rN_A$, or $\tau_p = 1/rN_D$,

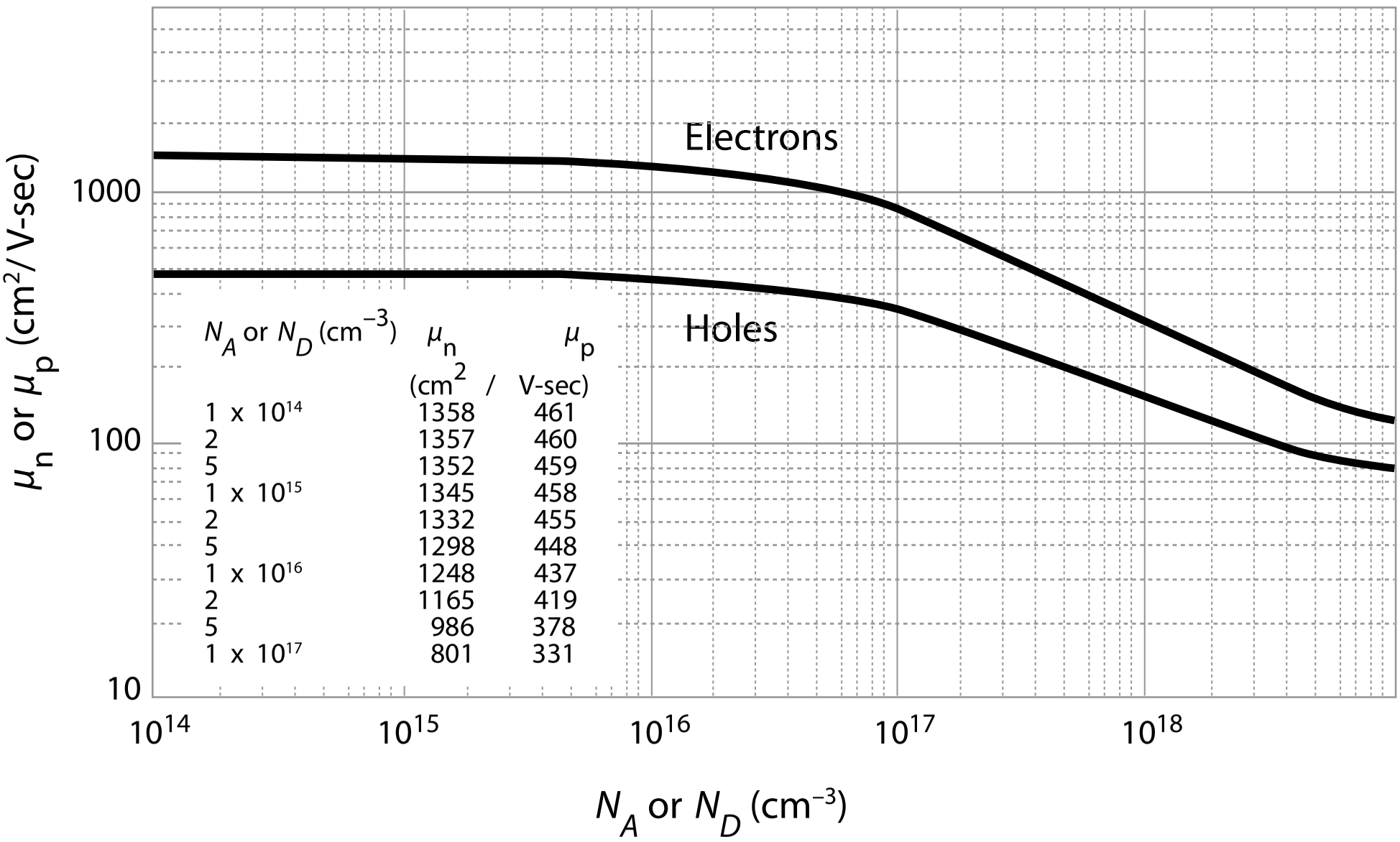
Minority carrier diffusion lengths are $\lambda_n = \sqrt{\tau_n D_n}$, or $\lambda_p = \sqrt{\tau_p D_p}$.

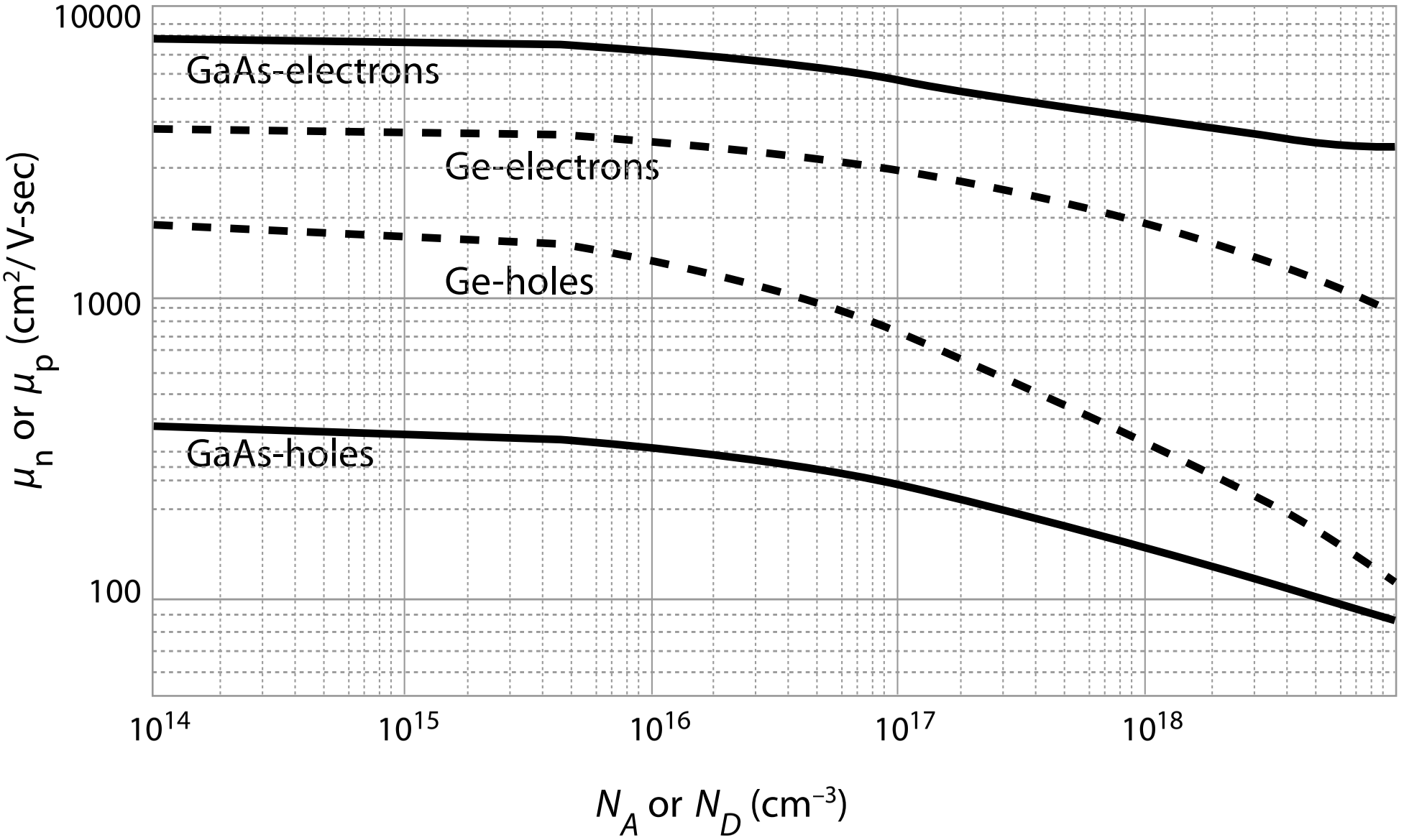
If traps dominate the recombination, then $\tau = 1/r_2 N_T$ where $r_2 >> r$


10^7
10^6
10^5
v_d (cm/sec)
Electrons
Holes
$v_d = v_{sat}$
$v_d \propto \mathscr{E}$
10^2
10^3
10^4
10^5
10^6
$\mathscr{E}$ (V/cm)


10^4
10^3
10^2
μ_n (cm^2/V-sec)
$N_D = 10^{14}$
$N_D = 10^{16}$
$N_D = 10^{17}$
$N_D = 10^{18}$
$T^{-2.3}$
Silicon
100
200
500
1000


Silicon
$N_A = 10^{14}$
$N_A = 10^{16}$
$N_A = 10^{17}$
$N_A = 10^{18}$
$T^{-2.2}$
μ_p (cm^2/V-sec)
10^4
10^3
10^2
100
200
500
1000


Silicon
T = 300 K
Electrons
Holes
1000
100
10
μ_n or μ_p (cm^2/V-sec)
10^{14}
10^{15}
10^{16}
10^{17}
10^{18}
N_A or N_D (cm^{-3})
N_A or N_D (cm^{-3}) μ_n μ_p
(cm^2 / V-sec)
1 x 10^{14} 1358 461
2 1357 460
5 1352 459
1 x 10^{15} 1345 458
2 1332 455
5 1298 448
1 x 10^{16} 1248 437
2 1165 419
5 986 378
1 x 10^{17} 801 331


T = 300 K
10000
1000
100
GaAs-electrons
Ge-electrons
Ge-holes
GaAs-holes
μ_n or μ_p (cm^2/V-sec)
10^{14}
10^{15}
10^{16}
10^{17}
10^{18}
N_A or N_D (cm^{-3})

USEFUL CARRIER MODELING EQUATIONS

Density of States and Fermi Function:

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, E \leq E_v$$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Carrier Concentration Relationships:

$$n = N_C e^{(E_F - E_C)/kT}$$

$$p = N_v e^{(E_v - E_F)/kT}$$

$$n = n_i e^{(E_F - E_i)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

$$N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2}$$

$$N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}$$

n_i, np-Product, and Charge Neutrality:

$$n_i = \sqrt{N_c N_v} e^{-E_G/2kT}$$

$$np = n_i^2$$

$$p - n + N_D - N_A = 0$$

n, p, and Fermi Level Relationships:

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$E_i = \frac{E_C + E_v}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$n \approx N_D$$

$$p \approx n_i^2 / N_D$$

$$N_D >> N_A, N_D >> n_i$$

$$E_F - E_i = kT \ln(n / n_i) = -kT \ln(p / n_i)$$

$$p \approx N_A$$

$$n \approx n_i^2 / N_A$$

$$N_A >> N_D, N_A >> n_i$$

$$E_F - E_i = kT \ln(N_D / n_i) \quad N_D >> N_A, N_D >> n_i$$

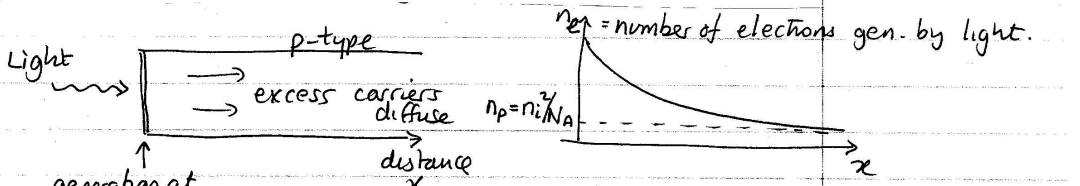
$$E_i - E_F = kT \ln(N_A / n_i)$$

Recombination and Generation Example

- At eqm, $R = rnp = rn_i^2$ for direct recombination. Therefore $G = R = rn_i^2$.
- Away from eqm. $R = rnp$, but n and p are non-eqm. concentrations.

For G, if the light is shining then $G \propto$ light intensity.
When the light is turned off, or inside the material, $G = n_i^2$

Example



N_A = doping level/cm³, n_p = eqm. # of electrons/cm³
n_e = # of electrons/cm³ gen. by light.

Always $$\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{drift} + \left.\frac{\partial n}{\partial t}\right|_{diffn.} + \left.\frac{\partial n}{\partial t}\right|_{REC} \quad (1)$$ Page 122

At steady state $\frac{\partial n}{\partial t} = 0$. In this case, all the G occurs at the surface. Inside the material, $G = G_{thermal} = rn_i^2 = rn_pN_A$

and $R = rnp \simeq rn_eN_A$ (since $n = n_p + n_e \approx n_e$)

hence $(R-G) = r(n_eN_A - n_pN_A)$

$\simeq rn_eN_A = n_e/\tau_n$ where $\tau_n = 1/rN_A$

Also, $E = 0$ inside material.

Hence $$\frac{\partial n}{\partial t} = 0 = \frac{1}{e}\left.\frac{\partial n}{\partial t}\right|_{diffusion} + -n_e/\tau_n$$

recall that $J_{diffusion} = eD_n\frac{\partial n}{\partial x}$ A/cm²

and $$\left.\frac{\partial n}{\partial t}\right|_{diff} = \frac{1}{e}\nabla J_{diff} = D_n\frac{d^2n}{dx^2}$$

So (1) becomes $0 = D_n\frac{d^2n}{dx^2} - n_e/\tau_n$ (with $n = n_p + n_e$; const, fn. of x)

which has a solution $$n_e = n_e(x=0)\exp^{-x/\sqrt{D_n\tau_n}} \quad (2)$$

General expressions for minority carriers

Minority carrier lifetime $\tau_p = 1/rN_D$ in n-type
$\tau_n = 1/rN_A$ in p-type

$$\left.\frac{\partial n}{\partial t}\right|_{thermal} = -\frac{n_e}{\tau_n}$$

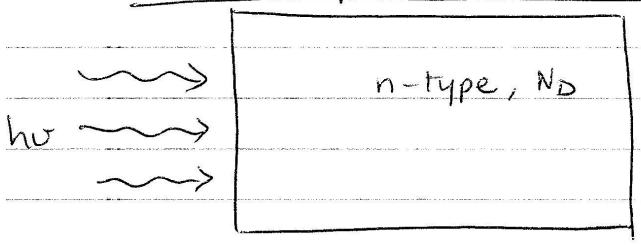
n_e, p_e are excess carrier concentrations (often written Δn, Δp)

$$\left.\frac{\partial p}{\partial t}\right|_{thermal} = -\frac{p_e}{\tau_p}$$

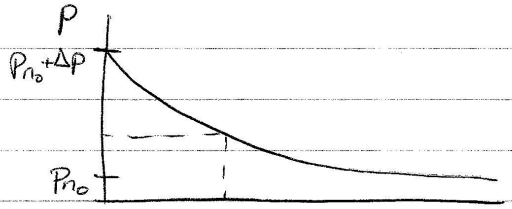
Diffusion lengths (before recombination occurs) are $\sqrt{D_n\tau_n}$ or $\sqrt{D_p\tau_p}$.

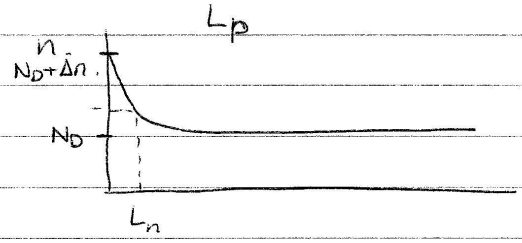
For recombination dominated by traps, use $\tau = 1/r_2N_T$, where $r_2 \gg r$. N_T is the trap density. (Indirect recombination). Page 112.

R&G example with different mobilities



Shine light on semiconductor
What is $n(x)$ and $p(x)$ if $\mu_n \neq \mu_p$?

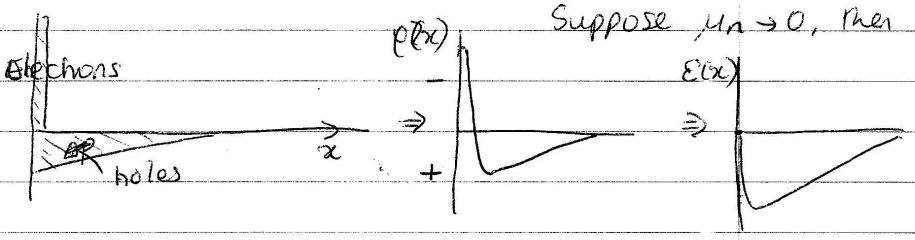




At $x=0$ $\Delta n = \Delta p$

Suppose $\mu_p \gg \mu_n$. $L_p \gg L_n$.

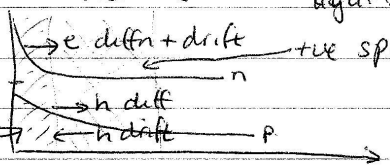
Expect holes to diffuse further before they recombine giving net -ve charge at small x and +ve at large x



Suppose $\mu_n \to 0$, then

Electric field pulls electrons → same direction as diffn
holes ← against diffusion

Steady state:
$\Delta n(x) \neq \Delta p(x)$



+ve space charge

-ve space charge