

Mathematics for Computer Science
 MIT 6.042J/18.062J


Simple Graphs: Coloring


 Albert R Meyer, April 5, 2013 coloring.1


Flight Gates

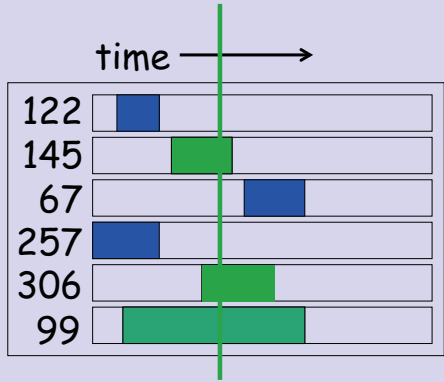
flights need gates, but
 times overlap.
 how many gates needed?


 Albert R Meyer, April 5, 2013 coloring.2



Airline Schedule


time →

Flights

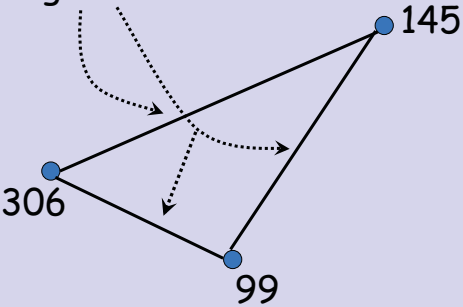



Flight	Start	End
122	0	10
145	10	20
67	20	30
257	0	10
306	10	20
99	0	30



 Albert R Meyer, April 5, 2013 coloring.3

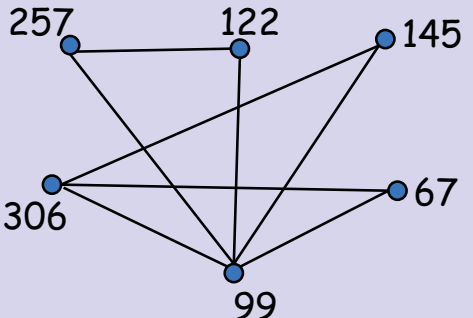

Conflicts Among 3 Flights


Needs gate at same time





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 **Model all Conflicts with a Graph**




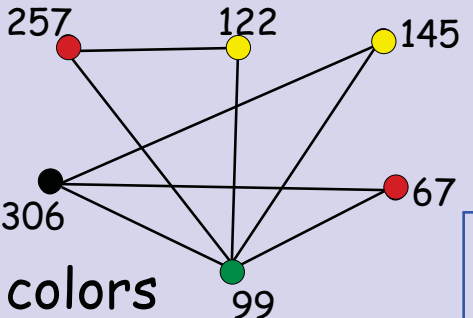
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 **Color the vertices**

Color vertices so that adjacent vertices have different colors.
 min # distinct colors needed =
 min # gates needed

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 **Coloring the Vertices**




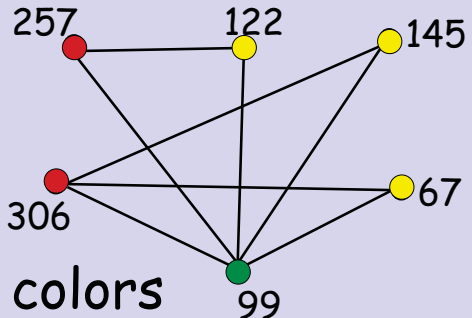
assign gates:

- 257, 67
- 122, 145
- 99
- 306


4 colors
4 gates


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 **Better coloring**





3 colors
3 gates

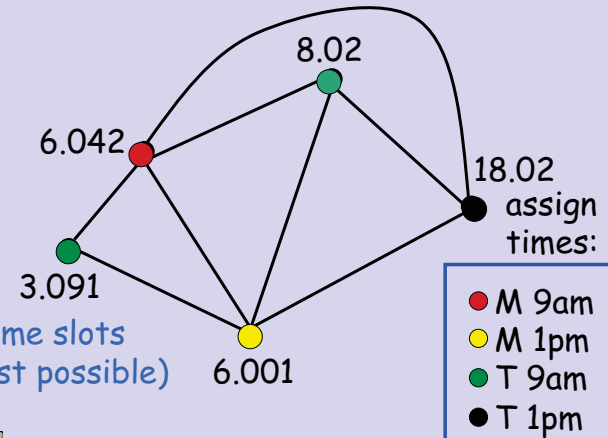
 Albert R Meyer, April 5, 2013 coloring.8

 **Final Exams**

subjects **conflict** if student takes both, so need different time slots.
how short an exam period?

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 **Model as a Graph**




assign times:


- M 9am
- M 1pm
- T 9am
- T 1pm


4 time slots (best possible)

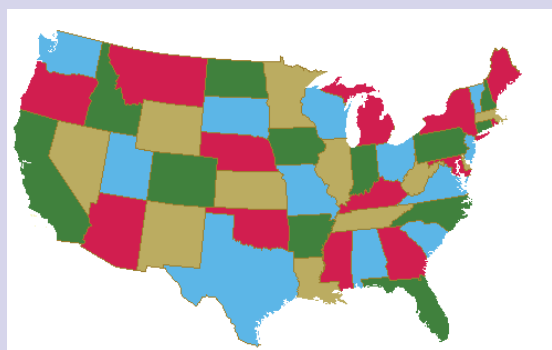
 Albert R Meyer, April 5, 2013 coloring.10


 **Conflicting Allocation Problems**

- # separate **habitats** to house different species of animals, some **incompatible** with others?
- # different **frequencies** for radio stations that **interfere** with each other?
- # different colors to **color a map**?

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 **Map Coloring**



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Countries are the Vertices

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Planar Four Coloring

any planar map is 4-colorable.
 1850's: false proof published
 (was correct for 5 colors).
 1970's: proof with computer
 1990's: much improved

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Chromatic Number

min #colors for G is
 chromatic number
 $\chi(G)$

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2


Simple Cycles

$\chi(C_{\text{even}}) = 2$


$\chi(C_{\text{odd}}) = 3$


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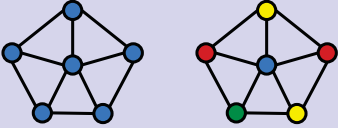

Complete Graph K_5



$\chi(K_n) = n$


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The Wheel W_n



W_5

$\chi(W_{\text{odd}}) = 4$
 $\chi(W_{\text{even}}) = 3$


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Bounded Degree

all degrees $\leq k$, implies


$\chi(G) \leq k+1$

very simple algorithm...

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"Greedy" Coloring

...color vertices in any order.
 next vertex gets a color
 different from its neighbors.
 $\leq k$ neighbors, so
 $k+1$ colors always work

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

coloring arbitrary graphs

2-colorable? --easy to check

3-colorable? --hard to check
(even if planar)

find $\chi(G)$? --theoretically
no harder than 3-color, but
harder in practice



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